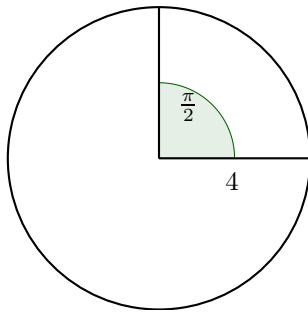


### Homework #3: radians, arcs, $\pi$

Chris DePauw, 02/19/09

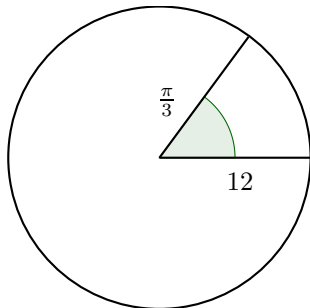
*find the exact length of each arc intercepted by the given central angle. (pg113)*

problem 1:



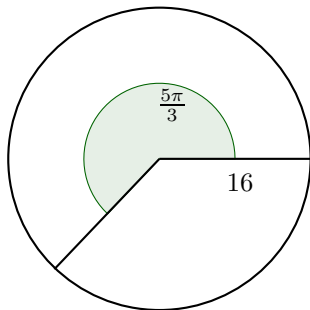
$$\begin{aligned}\theta &= \frac{s}{r} \\ \frac{\pi}{2} &= \frac{s}{4} \\ 4\pi &= 2s \\ 2\pi &= s \quad (\text{solved})\end{aligned}$$

problem 2:



$$\begin{aligned}\theta &= \frac{s}{r} \\ \frac{\pi}{3} &= \frac{s}{12} \\ 12\pi &= 3s \\ 4\pi &= s \quad (\text{solved})\end{aligned}$$

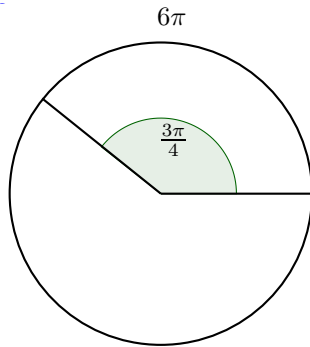
problem 3:



$$\begin{aligned}\theta &= \frac{s}{r} \\ \frac{5\pi}{3} &= \frac{s}{16} \\ 80\pi &= 3s \\ 20\pi &= s \quad (\text{solved})\end{aligned}$$

*Find the radius of each circle. (pg113)*

problem 4:



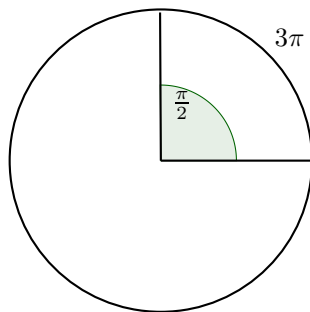
$$\theta = \frac{s}{r}$$

$$\frac{3\pi}{4} = \frac{6\pi}{r}$$

$$3\pi r = 24\pi$$

$$r = 8 \quad (\text{solved})$$

problem 5:



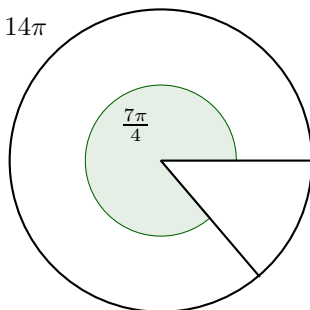
$$\theta = \frac{s}{r}$$

$$\frac{\pi}{2} = \frac{3\pi}{r}$$

$$\pi r = 2\pi$$

$$r = 6 \quad (\text{solved})$$

problem 6:



$$\theta = \frac{s}{r}$$

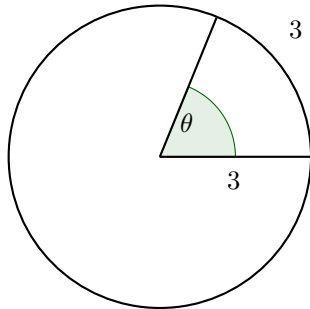
$$\frac{7\pi}{4} = \frac{14\pi}{r}$$

$$7\pi r = 56\pi$$

$$r = 8 \quad (\text{solved})$$

Find the measure of each central angle (in radians).  
(pg113)

problem 7:

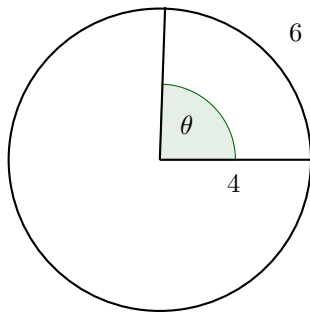


$$\theta = \frac{s}{r}$$

$$\theta = \frac{3}{3}$$

$$\theta = 1 \quad (\text{solved})$$

problem 8:

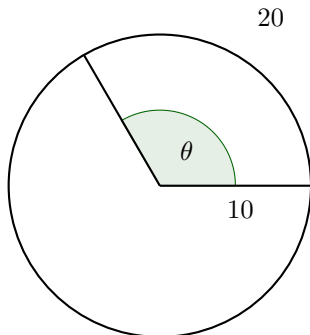


$$\theta = \frac{s}{r}$$

$$\theta = \frac{6}{4}$$

$$\theta = \frac{3}{2} \quad (\text{solved})$$

problem 9:



$$\theta = \frac{s}{r}$$

$$\theta = \frac{20}{10}$$

$$\theta = 2 \quad (\text{solved})$$

*Find the length to three significant digits of each arc intercepted by a central angle  $\theta$  in a circle of radius  $r$ . (pg113)*

problem 11:  $r = 12.3$  cm,  $\theta = \frac{2\pi}{3}$  radians

$$s = r\theta$$

$$s = 12.3 \text{ cm} \left( \frac{2\pi}{3} \right)$$

$$s = 25.761... \text{ cm} \quad (\text{use a calculator})$$

$$s = 25.8 \text{ cm} \quad (\text{solved})$$

problem 12:  $r = .892$  cm,  $\theta = \frac{11\pi}{10}$  radians

$$s = r\theta$$

$$s = .892 \text{ cm} \left( \frac{11\pi}{10} \right)$$

$$s = 3.082... \text{ cm} \quad (\text{use a calculator})$$

$$s = 3.08 \text{ cm} \quad (\text{solved})$$

problem 13:  $r = 1.38$  ft,  $\theta = \frac{5\pi}{6}$  radians

$$s = r\theta$$

$$s = 1.38 \text{ ft} \left( \frac{5\pi}{6} \right)$$

$$s = 3.6128... \text{ ft} \quad (\text{use a calculator})$$

$$s = 3.61 \text{ ft} \quad (\text{solved})$$

problem 14:  $r = 3.24$  mi,  $\theta = \frac{7\pi}{6}$  radians

$$s = r\theta$$

$$s = 3.24 \text{ mi} \left( \frac{7\pi}{6} \right)$$

$$s = 11.875... \text{ mi} \quad (\text{use a calculator})$$

$$s = 11.9 \text{ ft} \quad (\text{solved})$$

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**problem 15:**  $r = 4.82$  m,  $\theta = 60^\circ$

$$s = r\theta$$

$$s = 4.82 \text{ m } (60^\circ)$$

$$s = 4.82 \text{ m } \left(\frac{\pi}{3}\right) \quad (\text{convert to radian})$$

$$s = 5.0474\dots \text{ m} \quad (\text{use a calculator})$$

$$s = 5.05 \text{ m} \quad (\text{solved})$$

**problem 16:**  $r = 71.9$  cm,  $\theta = 135^\circ$

$$s = r\theta$$

$$s = 71.9 \text{ cm } (135^\circ)$$

$$s = 71.9 \text{ cm } \left(\frac{3\pi}{4}\right) \quad (\text{convert to radian})$$

$$s = 169.41\dots \text{ cm} \quad (\text{use a calculator})$$

$$s = 169 \text{ cm} \quad (\text{solved})$$

**problem 17:**  $r = 15.1$  in.,  $\theta = 210^\circ$

$$s = r\theta$$

$$s = 15.1 \text{ in. } (210^\circ)$$

$$s = 15.1 \text{ in. } \left(\frac{7\pi}{6}\right) \quad (\text{convert to radian})$$

$$s = 55.3443\dots \text{ in.} \quad (\text{use a calculator})$$

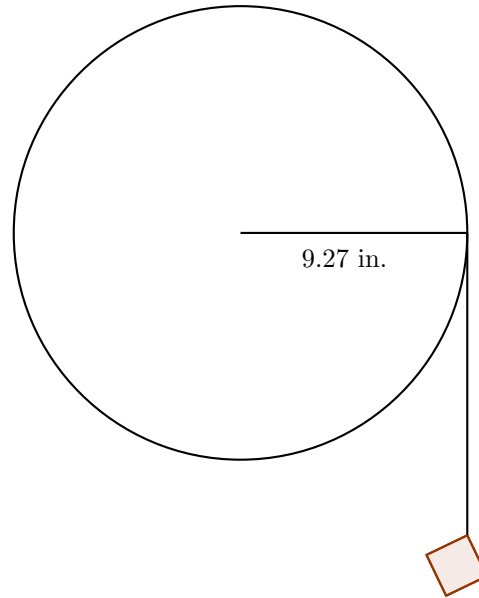
$$s = 55.3 \text{ in.} \quad (\text{solved})$$

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**work each problem (pg114)**

(a) How many inches will the weight in the figure rise if the pulley is rotated through an angle of  $71^\circ 50'$ ?

(b) Through what angle, to the nearest minute, must the pulley be rotated to raise the weight 6 in.?



$$71^\circ 50' = 71^\circ + \left(\frac{50}{60}\right)^\circ$$

$$71^\circ 50' = 71.8333\bar{3}^\circ$$

(convert all to degree)

$$71.8333\bar{3}^\circ = \left(\frac{71.8333\bar{3}\pi}{180}\right)$$

(convert to radian)

$$s = r\theta$$

$$s = 9.29 \left[71.833 \left(\frac{\pi}{180}\right)\right]$$

$$s \approx 11.6220\dots \quad (\text{use a calculator})$$

$$s \approx 11.6 \text{ in.} \quad (\text{solved})$$

$$\theta r = s$$

$$\theta \frac{\pi}{180} (9.27) = 6$$

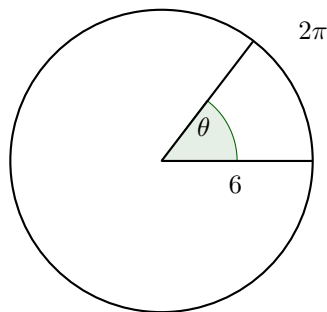
$$9.27\pi = 180 * 6$$

$$\theta = 37.0846\dots$$

$$\theta = 37^\circ 5' \quad (\text{solved})$$

**work each problem (pg114)**

**problem 39:**



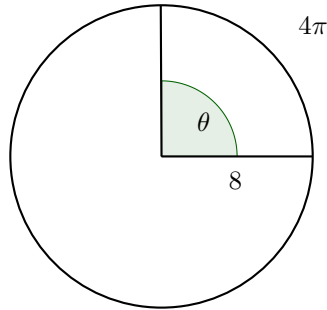
$$A = \frac{1}{2}r^2\theta \quad \theta = \frac{s}{r}$$

$$A = \frac{1}{2}(6)^2 \left(\frac{\pi}{6}\right)$$

$$A = 18 \left(\frac{\pi}{6}\right)$$

$$A = 6\pi \quad (\text{solved})$$

problem 40:



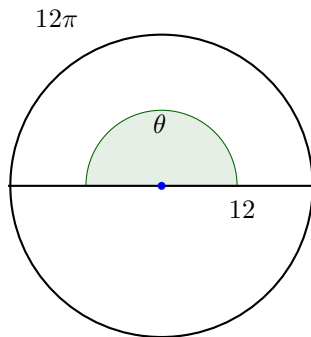
$$A = \frac{1}{2}r^2\theta \quad \theta = \frac{s}{r}$$

$$A = \frac{1}{2}(8)^2\left(\frac{4\pi}{8}\right)$$

$$A = \frac{1}{2} * 8 * 4\pi$$

$$A = 16\pi \quad (\text{solved})$$

problem 41:



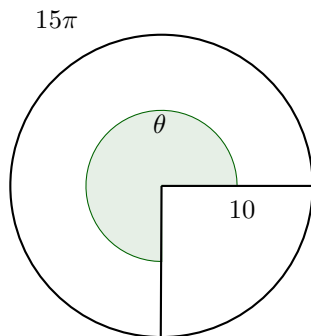
$$A = \frac{1}{2}r^2\theta \quad \theta = \frac{s}{r}$$

$$A = \frac{1}{2}(12)^2\left(\frac{12\pi}{12}\right)$$

$$A = \frac{1}{2}(12)(12\pi)$$

$$A = 72\pi \quad (\text{solved})$$

problem 42:



$$A = \frac{1}{2}r^2\theta \quad \theta = \frac{s}{r}$$

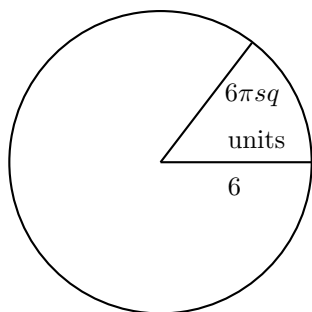
$$A = \frac{1}{2}(10)^2\left(\frac{15\pi}{10}\right)$$

$$A = \frac{1}{2}(10)(15\pi)$$

$$A = 75\pi \quad (\text{solved})$$

*Find the measure (in degrees) of each central angle. The number inside the sector is the area. (pg114)*

problem 43:



$$A = \frac{1}{2}r^2\theta$$

$$(6\pi) = \frac{1}{2}(6)^2\theta$$

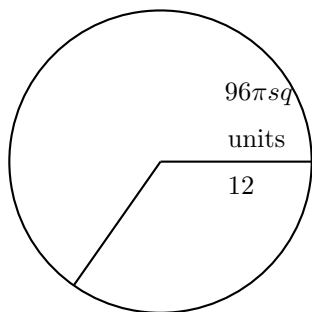
$$6\pi = \frac{1}{2}36\theta$$

$$\pi = 3\theta$$

$$\theta = \frac{\pi}{3}$$

$$\theta = 60^\circ \quad (\text{solved})$$

problem 44:



$$A = \frac{1}{2}r^2\theta$$

$$(96\pi) = \frac{1}{2}(12)^2\theta$$

$$96\pi = 72\theta$$

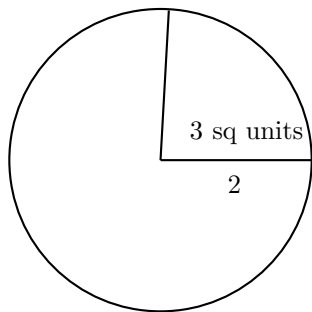
$$\theta = \frac{96\pi}{72}$$

$$\theta = \frac{4\pi}{3}$$

$$\theta = 30^\circ \quad (\text{solved})$$

*Find the measure (in radians) of each central angle. The number inside the sector is the area. (pg114)*

problem 45:



$$A = \frac{1}{2}r^2\theta$$

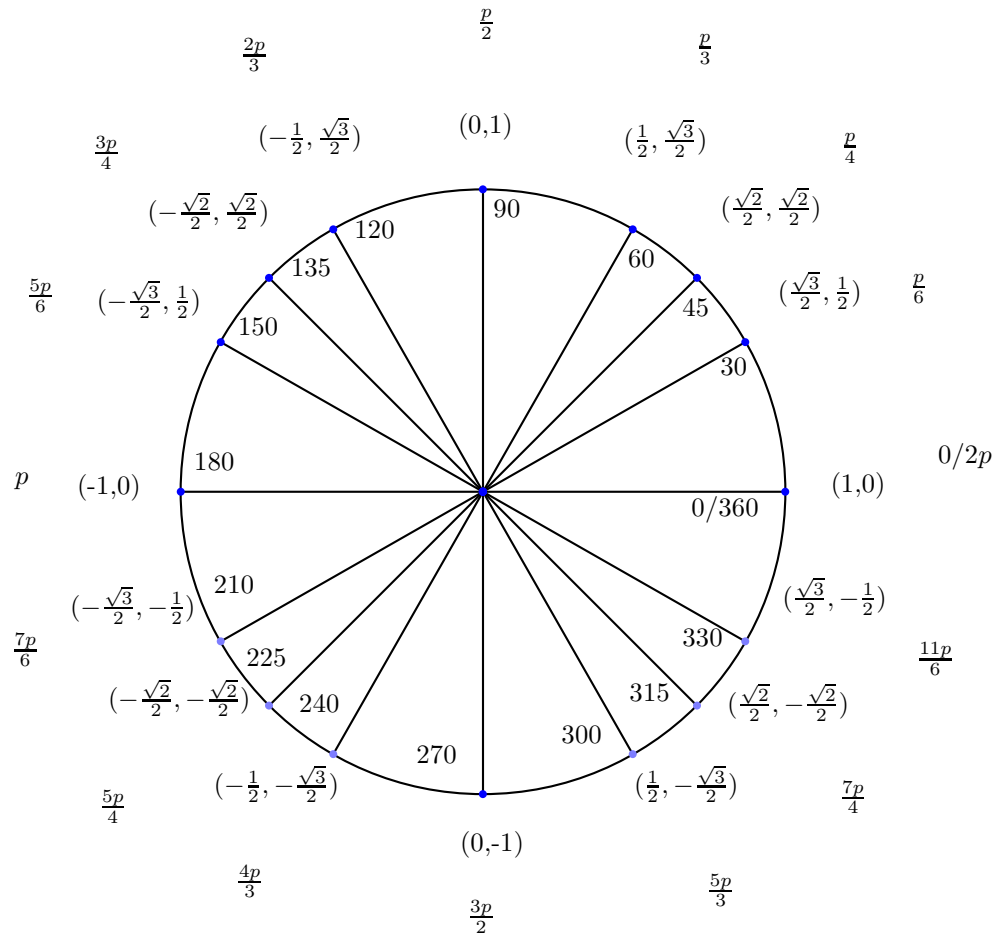
$$3 = \frac{1}{2}(2)^2\theta$$

$$3 = \frac{1}{2}4\theta$$

$$\theta = \frac{6}{4}$$

$$\theta = \frac{3}{2}$$

$$\theta = 1.5 \quad (\text{solved})$$



$$\sin s = y$$

$$\csc s = \frac{1}{y} \quad (y \neq 0)$$

$$\cos s = x$$

$$\sec s = \frac{1}{x} \quad (x \neq 0)$$

$$\tan s = \frac{y}{x} \quad (x \neq 0)$$

$$\cot s = \frac{x}{y} \quad (y \neq 0)$$

**For each value of the real number  $s$ , find (a)  $\sin s$ , (b)  $\cos s$  and (c)  $\tan s$ . (pg126)**

**problem 1:**  $s = \frac{\pi}{2}$

$$\begin{array}{lll} \sin s = 1 & (\sin s = y) & (0, 1) \\ \cos s = 0 & (\cos s = x) & \\ \tan s = ! & (\tan s = \frac{y}{x} [x \neq 0]) & \end{array}$$

**problem 2:**  $s = \pi$

$$\begin{array}{lll} \sin s = 0 & (\sin s = y) & (-1, 0) \\ \cos s = -1 & (\cos s = x) & \\ \tan s = 0 & (\tan s = \frac{y}{x} [x \neq 0]) & \end{array}$$

**problem 3:**  $s = 2\pi$

$$\begin{array}{lll} \sin s = 0 & (\sin s = y) & (1, 0) \\ \cos s = 1 & (\cos s = x) & \\ \tan s = 0 & (\tan s = \frac{y}{x} [x \neq 0]) & \end{array}$$

**problem 4:**  $s = 3\pi$

$$\begin{array}{lll} \sin s = 0 & (\sin s = y) & (-1, 0) \\ \cos s = -1 & (\cos s = x) & \\ \tan s = 0 & (\tan s = \frac{y}{x} [x \neq 0]) & \end{array}$$

**problem 5:**  $s = -\pi$

$$\begin{array}{lll} \sin s = 0 & (\sin s = y) & (-1, 0) \\ \cos s = -1 & (\cos s = x) & \\ \tan s = 0 & (\tan s = \frac{y}{x} [x \neq 0]) & \end{array}$$


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*Find the exact circular function value for each of the following. (pg126)*

**problem 7:**  $\sin \frac{7\pi}{6}$

$$\begin{aligned} \sin \frac{7\pi}{6} &= -\frac{\sqrt{2}}{2} && (\sin s = y) && \left( \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right) \\ \sin \frac{7\pi}{6} &= -\frac{\sqrt{2}}{2} && (\text{solved}) && \end{aligned}$$

**problem 8:**  $\cos \frac{5\pi}{3}$

$$\begin{aligned} \cos \frac{5\pi}{3} &= \frac{1}{2} && (\cos s = x) && \\ \cos \frac{5\pi}{3} &= \frac{1}{2} && (\text{solved}) && \left( \frac{1}{2}, -\frac{\sqrt{3}}{2} \right) \end{aligned}$$

**problem 9:**  $\tan \frac{3\pi}{4}$

$$\begin{aligned} \tan \frac{3\pi}{4} &= \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} && (\tan s = \frac{y}{x} [x \neq 0]) && \left( -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right) \\ \tan \frac{3\pi}{4} &= \frac{\sqrt{2}}{2} * -\frac{2}{\sqrt{2}} && && \\ \tan \frac{3\pi}{4} &= -1 && (\text{solved}) && \end{aligned}$$

**problem 10:**  $\sec \frac{2\pi}{3}$

$$\begin{aligned} \sec \frac{2\pi}{3} &= \frac{1}{-\frac{1}{2}} && (\sec s = \frac{1}{x} [x \neq 0]) && \left( -\frac{1}{2}, \frac{\sqrt{3}}{2} \right) \\ \sec \frac{2\pi}{3} &= 1 * -\frac{2}{1} && && \\ \sec \frac{2\pi}{3} &= -2 && (\text{solved}) && \end{aligned}$$


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